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Ultraslow optical waveguiding in an atomic Bose-Einstein condensate

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We investigate waveguiding of ultraslow light pulses in an atomic Bose-Einstein condensate. We show that under the conditions of off-resonant electromagnetically induced transparency, waveguiding with a few ultraslow modes can be realized. The number of modes that can be supported by the condensate can be controlled by means of experimentally accessible parameters. Propagation constants and the mode conditions are determined analytically using a WKB analysis. Mode profiles are found numerically. © 2008 Optical Society of America

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An optical pulse can propagate at ultraslow speeds through an atomic Bose-Einstein condensate (BEC)[1] under electromagnetically induced transparency (EIT)[2] conditions. Such pulses can be utilized for storage of coherent optical information.[3,4] Most of the early theoretical investigations assume one-dimensional light propagation,[5] except for a few notable studies.[6–8] Confinement of optical pulses in all three directions has been examined quite recently.[9] Transverse intensity profile of a focused control pulse translates to an effective, spatially varying refractive index for the probe pulse, which can be used for waveguiding and confinement purposes.[9] Controlled spatial variation of the refractive index profile was first suggested and used for creation of quantum shock waves with ultra-compressed slow light pulses.[6] The shock wave induced snake instability, depending on the transverse density

variations of the BEC, was observed.[6] More recently, transverse effects on slow light have been studied numerically.[10] It has been found that, using waveguiding by control pulses with Gaussian shaped transverse profiles, smaller group speeds can be achieved for higher order modes of the probe pulse.[10]

Possibility of slow light waveguiding in ultracold atomic medium has been demonstrated in a recent experiment.[11] Decrease of refractive index away from the optical axis and the corresponding graded index optical waveguiding are consequences of Gaussian density profile of the ultracold atomic cloud, strongly confined in transverse dimensions.[9] The experiment uses recoil induced resonance (RIR) method to achieve slow light ($c \sim 1500$ m/s), due to the strong dispersion in the high gain regime.[9] The ultracold atomic medium with RIR scheme operates as a graded index optical waveguide in the strongly guided regime due to the large core radius ($\sim 200\,\mu$m) and high refractive index contrast.[11]

In this letter, we investigate ultraslow waveguiding by BEC under EIT conditions. By considering slightly off-resonant EIT pulses propagating in a BEC, tightly trapped in transverse dimensions, possibility of weakly guided regime with few modes is shown. Single mode condition is established. These regimes are promising for guided nonlinear optical phenomena in ultracold matter.[11] Multiple modes at low temperatures are found. These results might be useful to design spatially controllable higher capacity optical memories.[10]

At low temperatures a Bose gas can be considered as a condensed cloud in a thermal gas background. Atomic number density profile of such a system can be described by[12] $\rho(\vec{r}) = \rho_c(\vec{r}) + \rho_{\text{th}}(\vec{r})$, where $\rho_{\text{c}}(\vec{r}) = [(\mu - V(\vec{r}))/U_0]\Theta(\mu - V(\vec{r}))$ is the number density of the condensed atoms and ρ_{th} is the density of the noncondensed ideal Bose gas. Here $U_0 = 4\pi\hbar^2 a_s/m$; m is atomic mass; a_s is the atomic s-wave scattering length. $\Theta(.)$ is the Heaviside step function and T_C is the critical temperature. The external trapping potential is $V(\vec{r}) = (m/2)(\omega_r^2 r^2 + \omega_z^2 z^2)$ with ω_r, ω_z are trap frequencies for the radial and axial directions, respectively. At temperatures below T_c, μ, the chemical potential is determined by $\mu(T) = \mu_{TF}(N_0/N)^{2/5}$, where μ_{TF} is the chemical potential evaluated under Thomas-Fermi approximation, $\mu_{TF} = ((\hbar\omega_t)/2)(15Na_s/a_h)^{2/5}$, with $\omega_t = (\omega_z\omega_r^2)^{1/3}$ and $a_h = \sqrt{\hbar/(\omega_z\omega_r^2)^{1/3}}$, the average harmonic oscillator length scale. The condensate fraction is given by $N_0/N = 1 - x^3 - s(\zeta(2)/\zeta(3))x^2(1-x^3)^{2/5}$, with $x = T/T_c$, and ζ is the Riemann-Zeta function. The scaling parameter s is given by $s = \mu_{TF}/k_B T_C = (1/2)\zeta(3)^{1/3}(15N^{1/6}a_s/a_h)^{2/5}$.

EIT susceptibility[2] for BEC of atomic density ρ can be expressed as $\chi = \rho\chi_1$ with

$$\chi_1 = \frac{|\mu|^2}{\epsilon_0\hbar}\frac{\mathrm{i}(-\mathrm{i}\Delta + \Gamma_2/2)}{[(\Gamma_2/2 - \mathrm{i}\Delta)(\Gamma_3/2 - \mathrm{i}\Delta) + \Omega_c^2/4]}, \tag{1}$$

where $\Delta = \omega - \omega_0$ is the detuning of the probe field frequency ω from the atomic resonance ω_0. Ω_c is the Rabi frequency of the control field; μ is the dipole matrix element for the probe transition. Γ_2 and Γ_3 denote the dephasing rates of the atomic coherence. We consider a

BEC of ^{23}Na atoms with parameters,[1] $N = 8.3 \times 10^6$, $\omega_r = 2\pi \times 69$ Hz, $\omega_z = 2\pi \times 21$ Hz, $\Gamma_3 = 0.5\gamma$, $\gamma = 2\pi \times 10.01$ MHz, and $\Gamma_2 = 2\pi \times 10^3$ Hz. We take $\Omega_c = 2.5\gamma$ and $\Delta = -0.1\gamma$ so that $\chi' = 0.04$ and $\chi'' = 0.0006$ at $T = 42$ nK. Here, χ' and χ'' are the real and imaginary parts of χ, respectively. Neglecting χ'', the refractive index becomes $n = \sqrt{1+\chi'}$. In the thermal cloud, $\chi' \ll 1$ so that an approximate refractive index profile in the radial direction can be written as

$$n(r) = \begin{cases} n_1[1 - A(\frac{r}{R})^2]^{1/2} & r \leq R. \\ 1 & r \geq R \end{cases}, \tag{2}$$

where $A = 1 - 1/n_1^2$ and $R = \sqrt{2\mu(T)/m\omega_r^2}$. In general n_1 is the index along the center line (z axis) of the condensate. n_1 is different than 1 only in the vicinity of the center of the condensate, where it varies slowly over the order of the optical wavelength. It can be determined by $n_1 = (1+\mu\chi_1'/U_0)^{1/2}$. In the radial direction, BEC density profile is translated to a refractive index analogous to a graded index fiber. Thermal gas background plays the role of the fiber coating while the condensate acts as the core.

We use the cylindrical coordinates as the refractive index $n(r)$ is axially symmetric. Small index difference between the condensed and noncondensed clouds results in weak guiding of probe pulse, for which the normal modes are the linearly polarized modes (LP modes)[13] which are determined by the Helmholtz radial equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + k_0^2 n^2(r) - \beta^2 - l^2/r^2\right]\psi(r) = 0. \tag{3}$$

Here $k_0 = \omega/c, l = 0, 1, 2, ...,$ β, and ψ are defined for the transverse field of the LP modes $E_t = \psi(r)\exp\left[\mathrm{i}(l\phi + \omega t - \beta z))\right]$. Employing Wentzel-Kramers-Brillouin (WKB) method, the characteristic equation is found to be

$$\int_{r_1}^{r_2} [k_0^2 n^2(r) - \beta_{lm}^2 - \frac{l^2}{r^2}]^{1/2}\mathrm{d}r = (m + \frac{1}{2})\pi, \tag{4}$$

where $r_{1,2}$ are the zeros of the integrand and $m = 0, 1, 2, 3, ...$ Solving Eq. 4 for β, we find

$$\beta_{lm} = n_1 k_0 [1 - \frac{2\sqrt{A}}{n_1 k_0 R}K]^{1/2}, \tag{5}$$

where $K = (l+2m+1)$. The propagation constant β_{lm}, depends on the frequency of the probe pulse. This frequency dependence determines the dispersion of the waveguide. Propagation constant can also be defined in terms of an effective mode index n_{lm} such that $\beta_{lm} = n_{lm}k_0 = n_{lm}(n_1, \omega)\omega/c$.

The group velocity v_g is defined as $1/v_g = \mathrm{d}\beta/\mathrm{d}\omega$, which leads to

$$\frac{1}{v_g} = \frac{d\beta}{d\omega} = \frac{\omega}{c}\left(\frac{\partial n_{lm}}{\partial n_1}\frac{\partial n_1}{\partial \omega} + \frac{\partial n_{lm}}{\partial \omega}\right) + \frac{n_{lm}}{c}. \tag{6}$$

Here, the terms in the parenthesis are due to the material and waveguide dispersion, respectively. Under EIT conditions, material dispersion dominates. Furhtermore, $n_{lm} \approx 1$ so that we can write

$$v_g = \frac{c}{1 + \omega \frac{\partial n_{lm}}{\partial n_1} \frac{\partial n_1}{\partial \omega}}. \tag{7}$$

The group velocity at $T = 42$ nK as a function of K is given in Fig.1. We calculate $v_g = 30.8$ m/s, for the LP_{00} mode. Group velocity increases with the mode number K. The higher order modes are away from the high density region of the condensed cloud and move faster. This behavior reduces modal dispersion and can be beneficial to preserve shape of the probe pulse for coherent optical information applications in BECs.

The dimensionless normalized frequency V is defined as $V = n_1 k_0 R \sqrt{A} = k_0 R (n_1^2 - 1)^{1/2}$. It determines the number of modes that can be supported by the atomic cloud. We find a confined mode description via $k_0 < \beta < n_1 k_0$ so that $K \leq V/2$. A condensate supports only single LP_{00} mode for $2 \leq V < 4$. For $2 \leq V < 6$, it could support just two modes, LP_{00} and LP_{10}. V can be tuned by various experimental parameters as $V = k_0 \mu (2\chi_1'/m\omega_r^2 u_0)^{1/2}$. Multiple-mode support can be perhaps most conveniently tuned by T. Temperature dependence of V is plotted in Fig.2. For the parameters used in Fig. 1, at $T = 42$ nK it starts from $V = 45$ and decreases down to a lowest value of $V \approx 7.3$ at $T = 398.7$ nK. As temperature increases, the condensed cloud shrinks and the index contrast diminish so that the BEC can no longer support multiple mode propagation.

More accurate results may be obtained using a numerical scheme. For that purpose it is assumed that the index of refraction increases incrementally from the edge to the center of the core. If the number of increments is high enough, this model will be a faithful representation of the conditions prevailing in the inhomogeneous core. Within each increment the refractive index is constant and we can use exact solutions of the wave equation in terms of Bessel functions. Electromagnetic boundary conditions provide a recurrence relation between successive solutions as well as an equation for the propagation constant. We have compared the results of our numerical calculation with these of WKB calculations and found excellent agreement. Typical results of our numerical simulations are given in Fig.3, where the modes LP_{00} and LP_{10} are shown. The modes are localized in the condensate core of radius $R = 21.1\,\mu$m. Beyond R, in the thermal component, the modes are evanescent. Such modes can be addressed by specifically constructed transverse profiles of the probe pulses. This can be exploited to control of capacity in the transverse direction for storage of coherent optical information in different mode patterns. These results can be further combined with the longitudinal control of optical information storage capacity where a particular EIT scheme, in which the control field is perpendicular to the probe field, is employed.[6]

Summarizing, we have examined optical waveguiding of ultraslow pulses in a Bose-Einstein

condensate under EIT conditions. Off-resonant EIT scheme has been considered to permit multi-mode ultraslow light propagation. Propagation constant is calculated analytically using WKB approximation. Mode profiles are determined numerically. Single and two mode conditions are established in terms of experimentally accessible parameters. Temperature dependence of the number of modes that can be supported by BEC is presented. Such ultraslow modes may be useful for realizing spatially controllable storage of coherent optical information with higher capacity in BECs, in the transverse direction, complementing longitudinal control of information storage capacity.[6]

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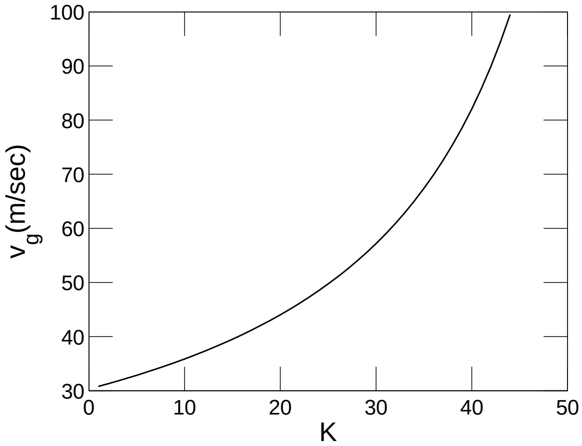

Fig. 1. Dependence of the group velocity on $K = (l + 2m + 1)$ at $T = 42$ nK ($T_c = 424$ nK). Parameters are given in the text. For the single mode condition group velocity is 30.8 m/s.

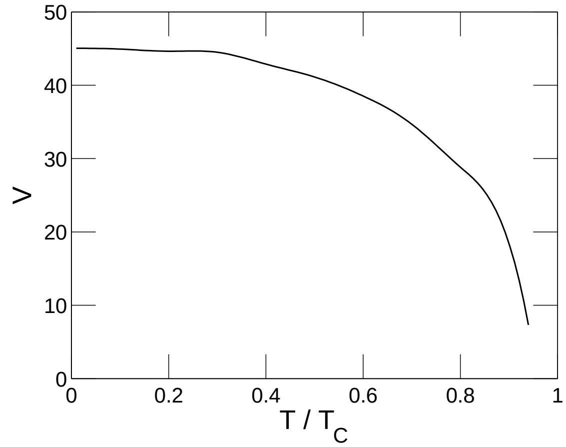

Fig. 2. Temperature dependence of V is plotted from $T = 42$ nK to $T = 398.7$ nK ($T_c = 424$ nK). V is related to K via $K \leq V/2$.

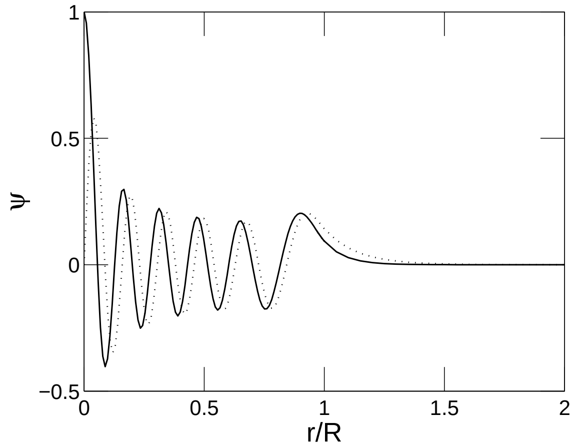

Fig. 3. Radial profiles of transverse modes LP_{00} (solid line) and LP_{10} (dotted line) confined in a BEC of ^{23}Na atoms with parameters given in the text.